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Factorization of Regge Slopes for Ordinary and New Hadrons

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ABSTRACT

We propose a factorization property of Regge slopes between ordinary and new hadrons. We derive three (six) relations for bosons (baryons). This is supported experimentally in a known case and several predictions are made. The D^{**} mass is predicted to be 2.36 GeV.

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Recent discoveries of new hadrons[1] strongly suggest a new degree of freedom (charm[2,3]) in particle physics. Hence we need a dynamics for these new hadrons. Before charm was found, it turned out to be very useful to combine the quark model with duality in order to construct a consistent hadron dynamics. Since charm has been found, there is a great need for a new hadron dynamics to accommodate charm in the duality scheme of hadrons.

There are similarities as well as differences between the ordinary and the new hadrons. One of the differences is that the Regge slopes of the new hadrons appear to be smaller than those of the ordinary ones if one approximates the ψ - χ trajectory by a straight line passing ψ and $\chi\,(2^+)$. As was argued by Mandelstam[4] some time ago, a four-point amplitude of the Veneziano type would lead to an increasing exponential behavior for large s even in the physical regions with fixed angles when the two Regge slopes are different in the s- and t-channels. This might throw some doubts on the possibility that the new hadrons could be related to ordinary hadrons through duality. As was discussed in a previous paper,[5] however, this difficulty could be overcome by taking a string picture of hadrons in which quark and anti-quark with finite masses are attached to a massless string, and by considering the rigid rotation of such a system.[6] If one considers[6] the ψ - χ Regge trajectory as the $c\bar{c}$ system, for example, the slope of the ψ - χ trajectory is small for low spins while it would approach a universal slope at high spins since the c-quark mass would be negligible compared with the energy of the string in that region. Thus, the slopes of the new and ordinary hadrons will both approach the universal slope at infinity even though these slopes are different in the resonance regions. Thus the new and old hadrons could be related through duality without contradicting Mandelstam's arguments.[4]

It is not yet known, however, how to construct an explicit dual model for arbitrary trajectories in practice even if it is possible in principle. Suppose we can approximate trajectories by straight lines at low and moderately high energies and consider the FESR duality connecting a sum of s-channel resonances to the t-channel Regge poles in these regions. A B_4 amplitude connecting any pair of trajectories in the s- and t-channels satisfies this duality. Expanding such an amplitude at moderately high energies and imposing the factorization property of Regge residues[7] we are then led to a factorization property of Regge slopes.

The purpose of this article is to propose such a new factorization property of Regge slopes relating ordinary and new hadrons. We derive three independent relations for boson slopes and six independent relations for baryon slopes, and then discuss several predictions as a consequence of our scheme.

For simplicity we first consider <u>meson-meson scattering</u>. Suppose we take the following reactions

$$D\bar{D} \to \rho \to D\bar{D}\ , \quad D\bar{D} \to \rho \to \pi\pi\ , \quad \pi\pi \to \rho \to \pi\pi$$

in the t-channel. Then the t-channel ρ-pole residues have to factorize[7]

$$\beta_{D\bar{D}\rho D\bar{D}}(t) \cdot \beta_{\pi\pi\rho\pi\pi}(t) = \left[\beta_{D\bar{D}\rho\pi\pi}(t)\right]^2 . \qquad (1)$$

Here we have defined the residue function $\beta_{a\bar{c}\rho\bar{b}d}(t)$ so that $\mathrm{Im}A_{ab\to cd}(s,t) \xrightarrow[s\to\infty]{} \beta_{a\bar{c}\rho\bar{b}d}(t)s^{\alpha_\rho(t)}$. For brevity, let us choose the $\pi^+\pi^-$ scattering amplitude to be

$$A^{\pi^+\pi^-}(s,t) = -\lambda B_4(\alpha_\rho(s), \alpha_\rho(t)) , \qquad (2)$$

where $\alpha_\rho(t)$ denotes the ρ - f trajectory function and $B_4(x,y) \equiv \dfrac{\Gamma(1-x)\Gamma(1-y)}{\Gamma(1-x-y)}$. Then we obtain $\beta_{\pi\pi\rho\pi\pi}(t) \sim \dfrac{\pi}{\Gamma(\alpha_\rho(t))}\, {\alpha_\rho'}^{\alpha_\rho(t)}$ up to a constant factor. Similar considerations for the $D\bar{D}$ and πD scattering amplitudes[5] immediately lead to $\beta_{D\bar{D}\rho D\bar{D}}(t) \sim \dfrac{\pi}{\Gamma(\alpha_\rho(t))}\, {\alpha_\psi'}^{\alpha_\rho(t)}$ and $\beta_{D\bar{D}\rho\pi\pi}(t) \sim \dfrac{\pi}{\Gamma(\alpha_\rho(t))}\, {\alpha_{D^*}'}^{\alpha_\rho(t)}$. Here $\alpha_\psi'(\alpha_{D^*}')$ denotes the slope for the $\psi - \chi(D^* - D^{**})$ trajectory, which is assumed to be linear in the resonance region. Substituting these relations into eq. (1), we are immediately led to

$$\frac{(\alpha_\psi')^{\alpha_\rho(t)}}{\Gamma(\alpha_\rho(t))} \cdot \frac{(\alpha_\rho')^{\alpha_\rho(t)}}{\Gamma(\alpha_\rho(t))} = \left[\frac{(\alpha_{D^*}')^{\alpha_\rho(t)}}{\Gamma(\alpha_\rho(t))}\right]^2 \qquad (3)$$

which must hold for any value of t. Therefore, we must have the relation

$$\alpha_\psi' \alpha_\rho' = (\alpha_{D^*}')^2 . \qquad (4)$$

One can easily show that this relation still holds even if the amplitudes include a finite number of satellite terms. Using the ρ-residue factorization for the reactions $D\bar{D} \to \rho \to D\bar{D}$, $D\bar{D} \to \rho \to \bar{K}K$ and $K\bar{K} \to \rho \to K\bar{K}$ as a second example, we obtain another new relation

$$\alpha_\psi' \cdot \alpha_\phi' = (\alpha_{F*}')^2 \quad . \tag{5}$$

Similar considerations for $K\bar{K} \to \rho \to K\bar{K}$, $K\bar{K} \to \rho \to \pi\pi$ and $\pi\pi \to \rho \to \pi\pi$ lead to the relation

$$\alpha_\phi' \cdot \alpha_\rho' = (\alpha_{K*}')^2 \quad . \tag{6}$$

The ϕ-residue factorization for $F\bar{F} \to \phi \to F\bar{F}$, $F\bar{F} \to \phi \to K\bar{K}$ and $K\bar{K} \to \phi \to K\bar{K}$ does not lead to a new relation but reduces to eq. (4). Similarly, the ψ factorization through $F\bar{F} \to \psi \to F\bar{F}$, $F\bar{F} \to \psi \to D\bar{D}$ and $D\bar{D} \to \psi \to D\bar{D}$ reduces to (6). Thus we have three independent relations without any inconsistency.

In order to test the above predictions, we use the following values for the Regge slopes (in $(\mathrm{GeV}/c)^{-2}$);

$$\begin{aligned} &\alpha_\rho' = \alpha'_{\rho - g} = 0.88, \quad \alpha_{K*}' = \alpha'_{K*-K**} = 0.82, \\ &\alpha_\phi' = \alpha'_{\phi - f'} = 0.79, \quad \alpha_\psi' = 0.50.^* \end{aligned} \tag{7}$$

The left-hand side of (6) is evaluated to be $(0.79)(0.88) = (0.83)^2$ which should be compared with $(\alpha_{K*}')^2 = (0.82)^2$. Thus, eq. (6) is well supported

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*See ref. 5.

experimentally. The other relations (4), (5) are difficult to test directly at present. Assuming eq. (4) to hold, however, one can deduce $\alpha_{D^*}' = 0.66$ using α_ρ' and α_ψ' as given in (7). If we assume $\alpha_{D^*}(s)$ to be linear and $M(D^*) = 2.01$ GeV,[1] we can predict the D** mass to be 2.36 GeV, which agrees with the value predicted by De Rújula, Georgi and Glashow.[8] This has to be checked experimentally. The relation (5) with α_ψ' and α_ϕ' as given in (7) also predicts $\alpha_{F^*}' = 0.63$. We show the ρ, K^*, ϕ, D^*, F^*, ψ trajectories as functions of $M^2 - M_V^2$ in fig. 1.

Let us now turn to a discussion of <u>meson-baryon scattering</u> with four flavors. Suppose we can write the (s, u) dual term for the invariant amplitudes A and B. The parity-conserving helicity amplitudes in the u-channel are expressed in terms of A and B, so similar discussions lead to factorization properties for baryon-Regge slopes. The results continue to hold with a finite number of satellite terms.*

Suppose we take the reactions $\pi N \to \Delta \to N\pi$, $\pi N \to \Delta \to \Sigma K$ and $K\Sigma \to \Delta \to \Sigma K$ in the u-channel. Then, the u-channel Δ-residue factorization gives us a relation

$$\alpha_{N^*}' \cdot \alpha_{\Xi^*}' = (\alpha_{Y^*}')^2 \tag{8}$$

between s-channel baryon slopes. This relation applies both to the octet and decuplet since each can couple to the u-channel Δ. If we twist final

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*Thus, the results still hold even when parity doublets in the resonance regions are eliminated by a finite number of satellite terms.

states, we obtain (6) between t-channel boson slopes. Similar considerations for the decuplet Y*, C*, Ω, Θ u-channel exchanges[9] with all possible combinations of external particles lead us to a set of results where in each case one of the three relations (4), (5), (6) holds between t-channel boson slopes and (8) or one of the following relations holds between u-channel baryon slopes:

$$\alpha_{Y*}' \cdot \alpha_{\Omega}' = (\alpha_{\Xi*}')^2 \quad , \tag{9}$$

$$\alpha_{N*}' \cdot \alpha_{X*}' = (\alpha_{C*}')^2 \quad , \tag{10}$$

$$\alpha_{C*}' \cdot \alpha_{\Theta}' = (\alpha_{X*}')^2 \quad , \tag{11}$$

$$\alpha_{C*}' \cdot \alpha_{T*}' = (\alpha_{S*}')^2 \quad , \tag{12}$$

$$\alpha_{Y*}' \cdot \alpha_{X_s*}' = (\alpha_{S*}')^2 \quad . \tag{13}$$

The Ξ*, X*, S*, T*, X_s* exchanges give no new information. We may visualize the various baryon slopes as in fig. 2 as a consequence of (8) ~ (13).

In conclusion, we emphasize that the accommodation of charm in the duality scheme would lead to many interesting predictions.

A more detailed presentation including further applications will be published elsewhere.

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FIGURE CAPTIONS

Fig. 1: The ρ, K^*, ϕ, D^*, F^*, ψ trajectories as a function of $M^2 - M_V{}^2$.

Fig. 2: The N^*, Y^*, Ξ^*, Ω, C^*, S^*, T^*, X^*, $X_s{}^*$, Θ trajectories as a function of $M^2 - \left(M_{3/2^*}\right)^2$.

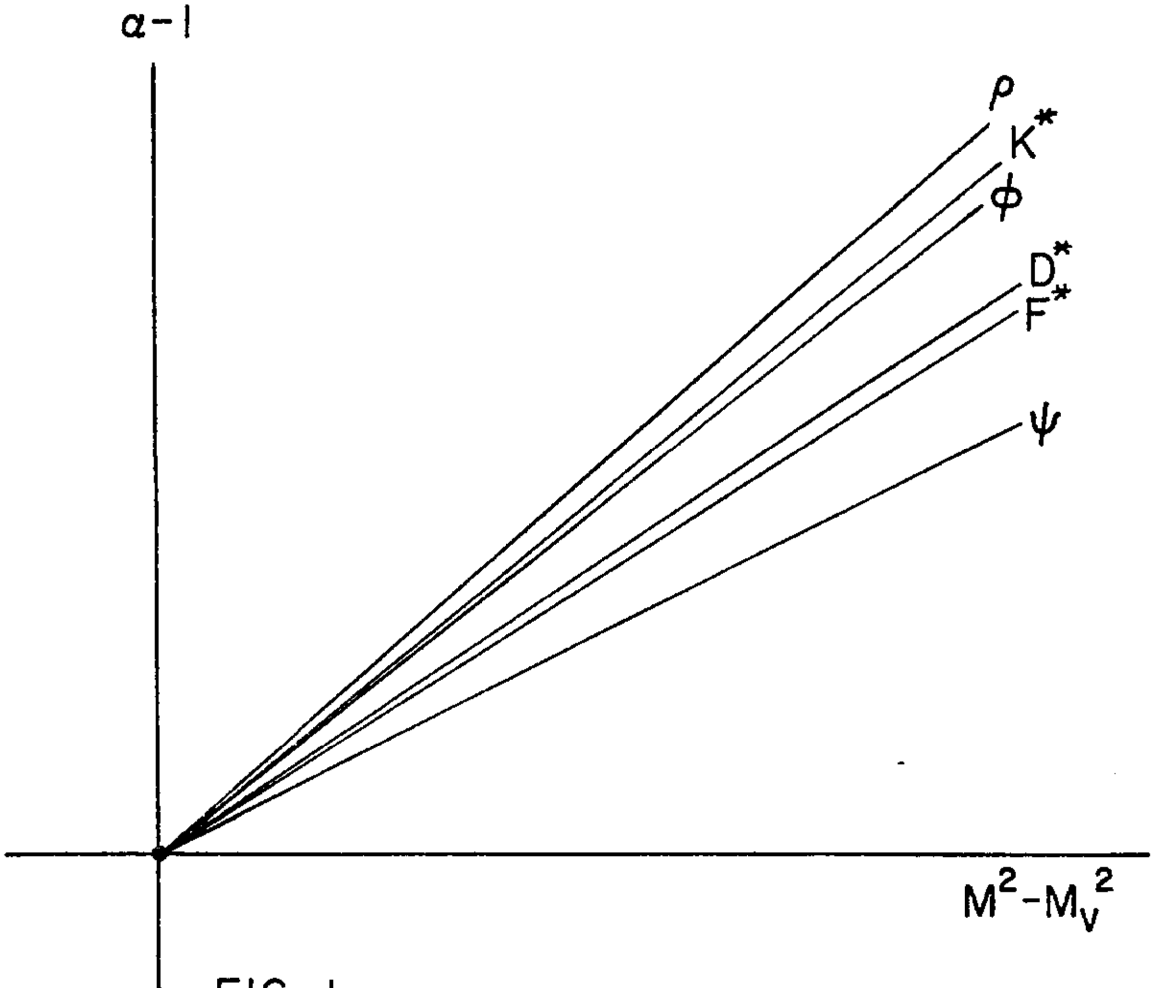

FIG. 1

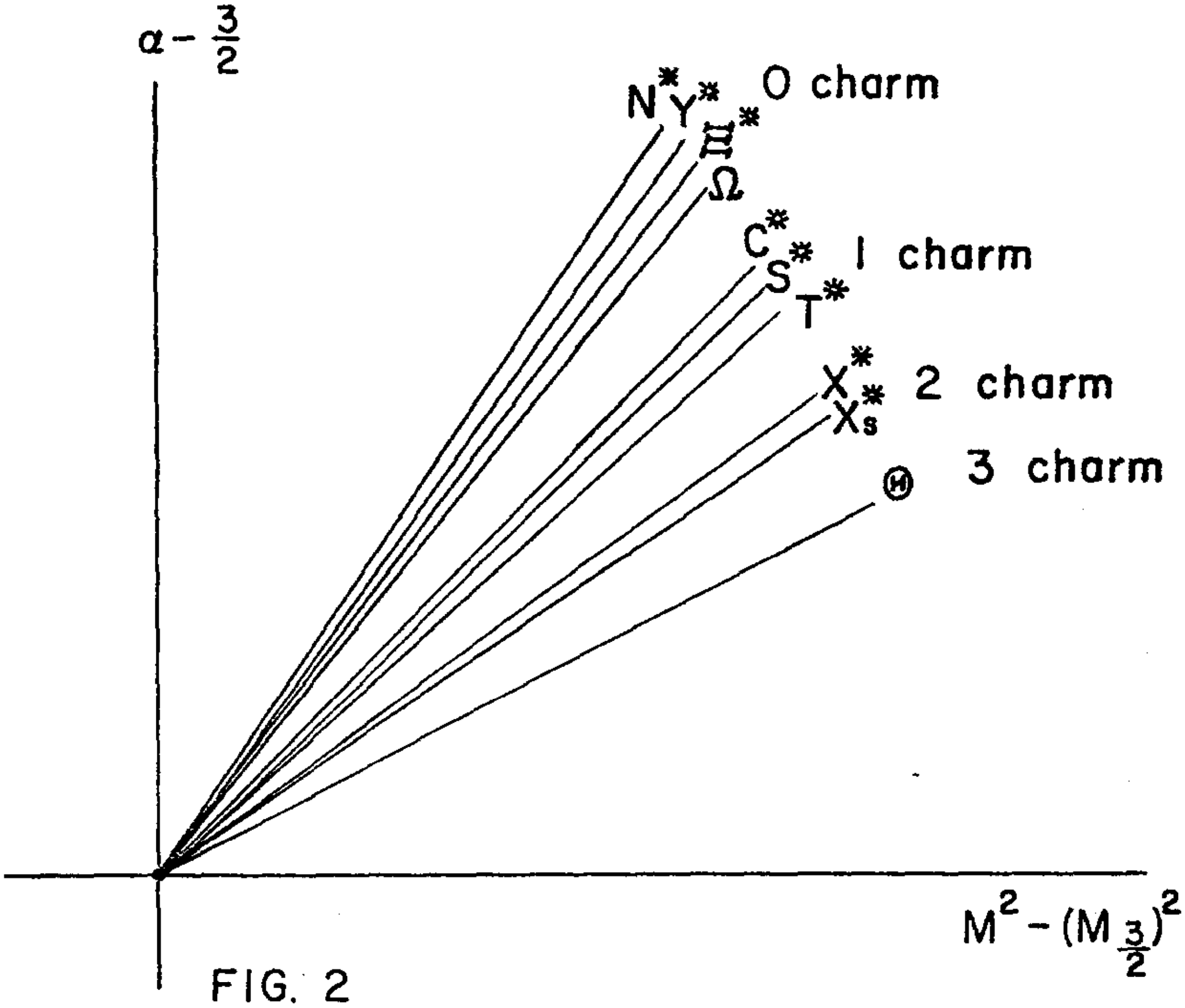

FIG. 2